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Mathematical model for heat transfer in variable thickness fins for rocket engines

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Purpose. This article aims to develop a mathematical model for a fin in the cooling system of liquid propellant rocket engines. The objective is to enable calculations for fins with arbitrary thickness variation. The developed mathematical model will be valuable and in demand for calculating heat transfer in the chambers of liquid propellant rocket engines produced using additive manufacturing technologies. **Design / Methodology / Approach**. The study employs theoretical research methods. The temperature distribution along the fin's height is derived by applying established heat transfer laws to the control volume under consideration. **Findings**. The study resulted in a mathematical model for a fin of variable thickness. The model was transformed into a dimensionless form to improve the accuracy of solving the equation numerically. Next, test calculations were performed using the proposed model. **Theoretical Implications**. This study builds upon existing models of heat transfer in fins and significantly extends the scope for further analysis by allowing for arbitrary variations in fin thickness. **Practical Implications.** The developed mathematical model can be applied to calculate the fin efficiency when designing cooling systems for combustion chambers, gas generators, and other components of liquid propellant rocket engines. **Originality / Value**. The article presents an original approach to calculating heat transfer in fins with variable thickness, enhancing its value for practical calculations. It can also serve as a reference for developing similar mathematical models. **Research Limitations / Future Research**. This study is focused on fins used in the cooling systems of liquid propellant rocket engine chambers. Therefore, the developed model is applicable only to fins where the longitudinal dimension significantly exceeds the transverse dimension. Future research could explore optimizing fin shapes to enhance heat transfer efficiency. **Article Type**. Applied Research.

Keywords:

variable thickness fin, mathematical model of heat transfer, liquid propellant rocket engine, cooling system of engine chamber, additive manufacturing, fin efficiency

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Over the years of rocket and space technology development, a robust theory of fin design has been established. However, the challenge of selecting the most optimal parameters for the cooling system remains unresolved (Leonardi et al., 2019; Desai & Kuzhiveli, 2022). This is due to the numerous factors that influence the process, as heat transfer through the finned surface depends on the geometric parameters of both the channel and the fin, the thermal conductivity of the wall material, the heat transfer coefficient, and more.

Furthermore, the authors of existing fin models were constrained by the manufacturing technologies available at the time, limiting the analysis to straight, triangular, and trapezoidal fin shapes. Recently, this situation has changed with the rise of a new and increasingly popular technology for producing liquid propellant rocket engine (LPRE) chambers – additive manufacturing (Vekilov et al., 2021). The development of technical capabilities for producing complex-shaped structures has created a demand for new modeling and analysis methods to fully leverage these advancements. A prime example of such technological progress is illustrated in the article (Bondarenko & Tkachov, 2024). The use of 3D printing effectively eliminates the limitations imposed by traditional manufacturing methods, such as milling, assembling tubular chambers, or using corrugated spacers. Therefore, this study will focus on ribs with variable thickness and arbitrary shapes.

Moreover, the modern advancements in computing technology eliminate the need for compact and simplified analytical expressions in calculations, enabling the use of more efficient numerical models (Алєксєєнко & Бучарський, 2024; Dubrovskiy & Bucharskyi, 2023).

The developed model is designed to complement the previously proposed mathematical model of the cooling system for liquid propellant rocket engine chambers (Бучарський & Слюсарєв, 2024). The presented approaches to heat transfer calculation can enhance cooling system efficiency. This improvement could help address issues related to thermal loads in efficient aerospike nozzles, as described in (Золотько, 2024).

In addition, the developed mathematical models are applicable not only to LPRE chambers but also to other components, such as gas generators, flow-cooled turbine stators, heat exchangers. For instance, reducing the mass and improving the efficiency of the heat exchanger's finned surface represent important steps for advancing pressurization system, as noted in (Мітіков & Седченко, 2023).

Objective and Tasks

The objective of this study is to develop a mathematical model for a LPRE cooling fin with an arbitrary thickness variation.

To achieve this objective, the following tasks must be completed:

– derive an equation for the temperature distribution along the fin's height based on established heat transfer laws, select appropriate boundary conditions, and reduce the equation to a dimensionless form;

– conduct test calculations to verify the accuracy of the developed model.

Materials and Methods

This study examines the cooling system of the LPRE chamber featuring a fin with a variable cross-section (see Fig. 1).

The effects of the combustion chamber and the outer wall will not be considered, and the symmetry of the fin allows us to analyze only half of it for the sake of computational convenience. For the portion of the fin under consideration, we will establish a Cartesian coordinate system, define the primary geometric dimensions, and designate an elemental volume, Δx (see Fig. 2).

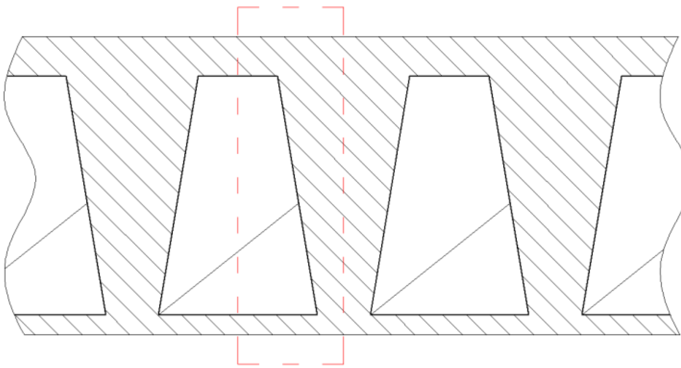

Figure 1 – Cooling channels of the LPRE chamber (Source: Authors)

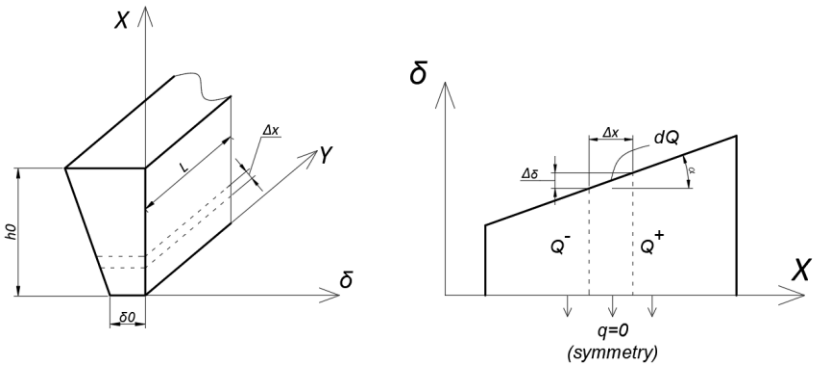

Figure 2 – Section of the fin under consideration (Source: Authors)

The study focuses on tall, thin fins made from materials with high thermal conductivity. This implies that temperature variations along the thickness of the fin can be neglected, making the fin temperature a function of its height. Additionally, it is assumed that the heat transfer coefficient between the fin and the coolant is known and remains constant along the entire height of the fin. The data used for the test calculations are presented in Table 1.

Table 1 – Initial data for calculating heat transfer in a fin (Source: Authors)

Parameter	Value
Half width of fin base, mm	0.5
Fin height, mm	3
Material thermal conductivity coefficient, Wt/m/K	300
Cooler heat transfer coefficient, Wt/m^2/K	20000
Fin base temperature, K	900
Cooler temperature, K	300

Like the approach taken in (Yang & Naraghi, 2020), numerical methods for solving differential equations were employed to calculate complex geometries using the model. In this study, the fourth-order Runge-Kutta method, implemented in the NDSolve function of the Wolfram Mathematica software, was utilized for this purpose.

Results

To develop the mathematical model, we apply the fundamental laws of heat transfer to the fin based on the established assumptions. According to Fourier's law, the heat flow through the left boundary of the control volume is given by:

$$Q^- = -\lambda^- \frac{dT^-}{dx} f^-,$$

where λ – thermal conductivity coefficient, T – temperature of the fin, f – cross-sectional area of the fin and x – coordinate along the height of the fin. The heat flow through the right boundary is calculated in a similar manner:

$$Q^+ = -\lambda^+ \frac{dT^+}{dx} f^+.$$

The amount of heat removed from the fin at the section Δx, according to the Newton equation, is given by:

$$dQ = \alpha \left(T - T_{liq}\right) S,$$

where S – lateral surface area of the fin. Therefore, the heat flow balance can be expressed as:

$$Q^- - dQ = Q^+,$$

can be rewritten as:

$$\lambda^+ \frac{dT^+}{dx} f^+ - \lambda^- \frac{dT^-}{dx} f^- = \alpha \left(T - T_{liq}\right) S. \tag{1}$$

In this case, the perimeter of the fin can be expressed as:

$$\Pi = 2\delta + L \approx L.$$

Then, the cross-sectional area is given by:

$$f = \delta\, L = \delta\, \Pi.$$

The lateral surface area is given by:

$$S = \Pi\, m,$$

where m – length of the generatrix, which can be expressed as follows:

$$m = \frac{\Delta x}{\cos(\alpha)},$$

where α – inclination angle of the fin (see Fig. 2). We can use the basic trigonometric identity to convert the known quantities:

$$\frac{1}{\cos(\alpha)} = \sqrt[2]{1 + tan^2(\alpha)} = \sqrt{1 + \left(\frac{\Delta\delta}{\Delta x}\right)^2}.$$

Then, the lateral surface area can be rewritten as:

$$S = \Pi\, \Delta x \sqrt{1 + \left(\frac{\Delta\delta}{\Delta x}\right)^2}.$$

By substituting the expressions for the geometric parameters into equation (1), we can divide both sides of the equation by $\Pi\, \Delta x$ to obtain:

$$\frac{\lambda^+ \frac{dT^+}{dx} f^+ - \lambda^- \frac{dT^-}{dx} f^-}{\Delta x} = \alpha\left(T - T_{liq}\right) \sqrt{1 + \left(\frac{\Delta\delta}{\Delta x}\right)^2}.$$

Let $\Delta x \to 0$ and take the limit:

$$\frac{d}{dx}\left(\lambda\, \delta\, \frac{dT}{dx}\right) = \alpha\left(T - T_{liq}\right) \sqrt{1 + \left(\frac{d\delta}{dx}\right)^2}. \tag{2}$$

To close this equation, we will apply the following boundary conditions: we will assume that the wall temperature at the base of the fin is specified as:

$$at\ x = 0 \to T(0) = T_0,$$

and that the heat flow through the top of the fin is zero:

$$at\ x = h_0 \to \frac{dT}{dx} = 0.$$

Since the equation will be solved numerically, it must be transformed into a dimensionless form to minimize calculation errors. The parameters α, h_0, T_0 were selected as the fundamental quantities, with all other quantities expressed in terms of these. Taking this into account,

$$\bar{\alpha} = \frac{\alpha}{\alpha} = 1;\ \bar{h}_0 = \frac{h_0}{h_0} = 1;\ \bar{T}_0 = \frac{T_0}{T_0} = 1;$$

$$\bar{x} = \frac{x}{h_0};\ \bar{\delta} = \frac{\delta}{h_0};\ \bar{T} = \frac{T}{T_0};\ \bar{T}_{liq} = \frac{T_{liq}}{T_0};\ \bar{\lambda} = \frac{\lambda}{\alpha\, h_0};$$

equation (2) can be expressed in dimensionless form as:

$$\frac{d}{d\bar{x}}\left(\bar{\lambda}\,\bar{\delta}\,\frac{d\bar{T}}{d\bar{x}}\right) = \left(\bar{T} - \bar{T}_{liq}\right)\sqrt{1 + \left(\frac{d\bar{\delta}}{d\bar{x}}\right)^2}\;;$$

$$\text{at } \bar{x} = 0 \rightarrow \bar{T}(0) = 1;$$

$$\text{at } \bar{x} = 1 \rightarrow \frac{d\bar{T}}{d\bar{x}} = 0.$$

To verify the derived equation, test calculations of the temperature distribution in the fin were performed. Three variations of the fin shape were considered during the calculations:

1. straight fin;
2. wavy fin;
3. fin with a smooth thickening toward its base.

The results of solving this equation, using the initial data provided in Table 1 for the selected fin shapes, are presented in Figure 3.

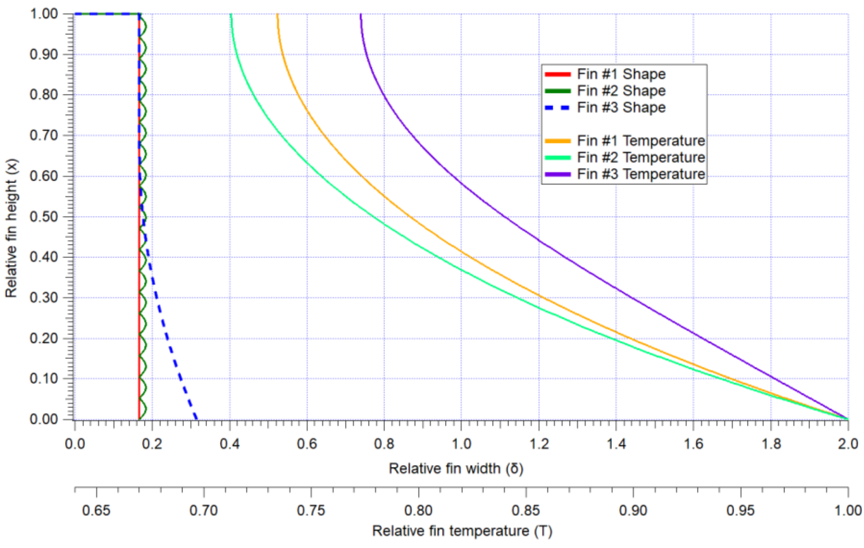

Figure 3 – Calculation Results Based on the Proposed Model (Source: Authors)

As observed, the calculation results align with theoretical concepts regarding heat transfer processes in fins. For instance, as the lateral surface area of the fin increases (due to its waviness), the heat transfer process becomes more efficient, resulting in a decrease in the temperature at the top of the fin. Conversely, when there is a local thickening at the base of the fin, the amount of heat transferred to the fin increases, leading to a rise in the temperature at the top of the fin.

Conclusions

In this study, a novel mathematical model of a fin with an arbitrary thickness variation was developed. To achieve this, a differential equation for temperature distribution along the height of the fin was derived based on established heat transfer laws. Subsequently, appropriate boundary conditions were selected, and the equation was transformed into a dimensionless form for numerical solution.

Test calculations were also performed using the developed model, confirming its accuracy.

Future work on this model can focus on refining the heat transfer processes in the fins. This may include accounting for variations in the material's thermal conductivity with changes in temperature, calculating the temperature distribution across the thickness of the fin, and conducting optimization calculations for fin design.

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